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New localized structures of a (2+1)-dimensional system obtained by variable separation approach

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Abstract. Starting from a quite universal formula, which is obtained by variable separation approach and valid for many (2+1)-dimensional nonlinear physical models, a new general type of solitary wave, i.e., semifolded solitary waves (SFSWs) and semifoldons, is defined and studied. We investigate the behaviors of the interactions for the new semifolded localized structures both analytically and graphically. Some novel features or interesting behaviors are revealed.

PACS. 05.45.Yv Solitons – 02.30.Jr Partial differential equations – 02.30.Ik Integrable systems

1 Introduction

In nonlinear science, soliton theory plays an essential role and has been applied in almost all natural sciences especially in all physics branches such as condensed matter physics, field theory, fluid dynamics, plasma physics, optics, etc. [1]. Most of the previous studies on soliton theory especially in higher dimensions are restricted to the single valued situations, such as dromion, compacton, peakon and their interactions, although there are some reports on multiple valued solitary waves (folded in all directions) [2–5]. However, our nature is colorful and may exhibit quite complicated multiple valued structures such as semifolded ones. For example, some localized structures such as ocean waves may fold in one direction, say x, and localize in a usual single valued way in another direction, say y. For convenience later, we define above localized excitations as semifolded solitary waves (SFSWs). Furthermore, if the interactions among the semifolded solitary waves are completely elastic, we call them semifoldons. In our knowledge, the interactions among single valued and multivalued (semifolded) localized excitations for (2+1)-dimensional integrable system were not reported in the previous literature. To study the interaction behaviors among them more direct and visually, we take a new (2+1)-dimensional nonlinear evolution equation, discussed by Maccari [6] by suitably utilizing the arbitrary functions presented in the system, as a concrete example. The system is of the form

$$i\phi_t + \phi_{xx} + \chi\phi = 0, \qquad (1a)$$

$$i\theta_t + \theta_{xx} + \chi\theta = 0, \qquad (1b)$$

$$\chi_y = \left(|\phi|^2 + |\theta|^2\right)_x, \qquad (1c)$$

where $\phi(x, y, t)$, $\theta(x, y, t)$ are complex and $\chi(x, y, t)$ is real. equations (1) are derived from Nizhnik equations through the reduction method. Uthayakunar et al. [7] have established the integrability property of equations (1) by using singularity structure analysis. Lai and Chow [8] obtained the generalized dromion solution and two-dromion solution of equations (1). Starting from a special Bäcklund transformation, we convert the equations (1) into simple variable separation equation, then obtain a quite general variable separation solution. Some types of the usual localized excitations of equations (1), such as dromions, lumps, ring soliton and oscillated dromion, breathers solution, etc, can be easily constructed by selecting appropriate arbitrary functions. Here, we only list some new and interesting localized excitations for equations (1). In particular, we are interested in the possible interaction behavior among localized excitations.

2 Variable separation solutions for the new (2+1)-dimensional nonlinear equation

To find out the interesting localized structures of the new equation system (1), first, we take the following Bäcklund transformation

$$\phi = \frac{g}{f} + \phi_0, \quad \theta = \frac{h}{f} + \theta_0, \quad \chi = 2(\ln f)_{xx} + \chi_{0x}, \qquad (2)$$

where f is a real, g and h are complex, and $(\phi_0, \theta_0, \chi_0)$ is an arbitrary seed solution. Under the transformation (2), equations (1) are transformed to their bilinear form,

$$(D_x^2 + iD_t)f \cdot g + \phi_0 D_x^2 f \cdot f + fg\chi_{0x} + f^2\phi_0\chi_{0x} = 0, \qquad (3)$$

$$(D_x^2 + iD_t)f \cdot h + \theta_0 D_x^2 f \cdot f + fh\chi_{0x} + f^2\theta_0\chi_{0x} = 0, \qquad (4)$$

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$$D_xD_yf\cdot f-gg^*-hh^*+f^2\chi_{0y}-fg\phi_0^*-fg^*\phi_0^*-fh\theta_0^*\\ -fh^*\theta_0^*-f^2(\phi_0\phi_0^*+\theta_0\theta_0^*)=0, \quad (5)$$

where D is the usual bilinear operator.

To discuss further, we fix the seed solution (ϕ_0,θ_0,χ_0) as

$$\phi_0=0,\theta_0=0,\chi_0=\chi_0(x,t), \quad (6)$$

then equations (3), (4) and (5) can be simplified to

$$(D_x^2+iD_t)f\cdot g+fg\chi_{0x}=0, \quad (7)$$

$$(D_x^2+iD_t)f\cdot h+fh\chi_{0x}=0, \quad (8)$$

$$D_xD_yf\cdot f=gg^*+hh^*. \quad (9)$$

To solve the bilinear equations (7) and (8) with equation (9), we make the ansatz

$$f=a_1p(x,t)-a_2q(y,t)+a_3p(x,t)q(y,t), \quad (10)$$

$$g=p_1(x,t)q_1(y,t)\exp\Big(ir_1(x,t)+is_1(y,t)\Big), \quad (11)$$

$$h=p_2(x,t)q_2(y,t)\exp\Big(ir_2(x,t)+is_2(y,t)\Big), \quad (12)$$

where a_1,a_2, and a_3 are arbitrary constants and $p,q,p_1,q_1,p_2,q_2,r_1,s_1,r_2,s_2$ are all real functions of the indicated variables. For simplicity, we choose $r_1=r_2=r$ in equations (11) and (12). Similar to the solving process of reference [9], we finally obtain

$$\phi=\frac{\varepsilon_1\varepsilon_2\sqrt{\lambda a_1a_2p_xq_y}\exp(ir+is_1)}{a_1p-a_2q+a_3pq}, \quad (13)$$

$$\theta=\frac{\varepsilon_3\varepsilon_4\sqrt{(2-\lambda)a_1a_2p_xq_y}\exp(ir+is_2)}{a_1p-a_2q+a_3pq}, \quad (14)$$

$$\chi=2\left(\frac{a_1p_{xx}+a_3p_{xx}q}{a_1p-a_2q+a_3pq}-\frac{(a_1p_x+a_3p_xq)^2}{(a_1p-a_2q+a_3pq)^2}\right)+\chi_{0x}, \quad (15)$$

where $p=p(x,t)$ is an arbitrary function of (x,t) thanks to the arbitrariness of the introduced seed function $\chi_0=\chi_0(x,t)$,

$$\chi_{0x}=(4p_x^2)^{-1}\left(4r_tp_x^2+4p_x^2r_x^2+p_{xx}^2-2p_xp_{xxx}\right), \quad (16)$$

where $q=q(y,t)$ is an arbitrary function of (y,t) satisfying the following Riccati equation,

$$q_t=-c_3(a_1+a_3q)^2-c_2(a_1+a_3q)+a_1a_2c_1, \quad (17)$$

and $r=r(x,t)$ is related to p with

$$p_t+2p_xr_x=c_1(-a_2+a_3p)^2+c_2(-a_2+a_3p)-a_1a_2c_3, \quad (18)$$

$s_i=s_i(y)(i=1,2)$ are arbitrary functions of y satisfying $s_{it}=0$ with λ,a_1,a_2, and a_3 being arbitrary constants, $c_i=c_i(t)(i=1,2,3)$ and $\varepsilon_1^2=\varepsilon_2^2=\varepsilon_3^2=\varepsilon_4^2=1$. Especially, the module square of the field ϕ and θ read

$$\Phi=|\phi|^2=\frac{\lambda a_1a_2p_xq_y}{(a_1p-a_2q+a_3pq)^2}, \quad (19)$$

$$\Theta=|\theta|^2=\frac{(2-\lambda)\,a_1a_2p_xq_y}{(a_1p-a_2q+a_3pq)^2}. \quad (20)$$

Because of the arbitrariness of the functions $p(x,t)$, $s_1(y)$, $s_2(y)$, and $c_i(t)(i=1,2,3)$, equations (19) and (20) reveal quite abundant soliton structures. Actually, from equations (19) and (20), it is easy to know the arbitrary p and q with the boundary conditions

$$p\,|_{x\to-\infty}\to B_1,p\,|_{x\to+\infty}\to B_2,\\ q\,|_{y\to-\infty}\to B_3,q\,|_{y\to+\infty}\to B_4, \quad (21)$$

where B_1,B_2,B_3, and B_4 are arbitrary constants which may be infinities, are coherent soliton solution localized in all directions. In the next section, we take physical quantity Φ as an example to discuss some interesting properties.

3 Some novel localized structures for the (2+1)-dimensional system equations (1)

It is interesting that the expression (19) is valid for many (2+1)-dimensional models like the DS equation, NNV system, ANNV equation and the BK equation, etc. [2–5,9,10]. Moreover, because of the arbitrariness of the functions p and q, included in (19), the quantity Φ possesses quite rich structures. For instance, if we select the functions p and q appropriately, we can obtain many kinds of localized solutions, like the dromions, lumps, ring soliton and oscillated dromion, breathers solution, fractal-dromion and fractal-lump soliton structures [2,10]. In addition to the usual localized structures, some new localized excitations like peakon, compacton, folded solitary wave and foldon solutions of equations (1) are found by selecting some types of lower-dimensional appropriate functions [2–5]. The properties of peakon-peakon, dromion-dromion, compacton-compacton, and foldon-foldon interactions were discussed in references [2–5,10]. In reference [9], we investigate the interactions among different types of solitary waves like peakons, dromions, and compactons both analytically and graphically. Now we pay our attention to the new semi-folded localized structures and interactions of single valued and multivalued (semifolded) localized excitations.

3.1 Asymptotic behaviors of the localized excitations produced from (19)

In general, if the function p and q are selected as localized solitonic excitations with

$$p\Big|_{t\to\mp\infty}=\sum_{i=1}^{M}p_i^{\mp},\quad p_i^{\mp}\equiv p_i(x-c_it+\delta_i^{\mp}), \quad (22)$$

$$q\Big|_{t\to\mp\infty}=\sum_{j=1}^{N}q_j^{\mp},\quad q_j^{\mp}\equiv q_j(y-C_jt+\Delta_j^{\mp}), \quad (23)$$

$$\Phi\left|_{t\to\mp\infty}\right. \to \sum_{i=1}^{M}\sum_{j=1}^{N}\left\{\frac{\lambda a_1 a_2 p_{ix}^{\mp} q_{jy}^{\mp}}{\left(a_1\left(p_i^{\mp}+P_i^{\mp}\right)-a_2\left(q_j^{\mp}+Q_j^{\mp}\right)+a_3\left(p_i^{\mp}+P_i^{\mp}\right)\left(q_j^{\mp}+Q_j^{\mp}\right)\right)^2}\right\}$$

$$\equiv \sum_{i=1}^{M}\sum_{j=1}^{N}\Phi_{ij}^{\mp}\left(x-c_i t+\delta_i^{\mp}, y-C_j t+\Delta_j^{\mp}\right)\equiv \sum_{i=1}^{M}\sum_{j=1}^{N}\Phi_{ij}^{\mp}, \quad (24)$$

where $\{p_i, q_j\}\,\forall i$ and j are localized functions, then the physical quantity Φ expressed by equation (19) delivers $M\times N$ (2+1)-dimensional localized excitations with the asymptotic behaviour

See equation (24) above

where

$$P_i^{\mp}=\sum_{j<i} p_j(\mp\infty)+\sum_{j>i} p_j(\pm\infty), \quad (25)$$

$$Q_i^{\mp}=\sum_{j<i} q_j(\mp\infty)+\sum_{j>i} q_j(\pm\infty), \quad (26)$$

and we have assumed without loss of generality, $C_i > C_j$ and $c_i > c_j$ if $i > j$.

It can be deduced from expression (24) that the ijth localized excitation Φ_{ij} preserves its shape during the interaction iff

$$P_i^{+}=P_i^{-}, \quad (27)$$

$$Q_j^{+}=Q_j^{-}. \quad (28)$$

Meanwhile, the phase shift of the ij th localized excitation Φ_{ij} reads

$$\delta_i^{+}-\delta_i^{-} \quad (29)$$

in the x direction and

$$\Delta_j^{+}-\Delta_j^{-} \quad (30)$$

in the y direction.

The above discussions demonstrate that localized solitonic excitations for the universal quantity Φ can be constructed without difficulties via the (1+1)-dimensional localized excitations with the properties (22), (23), (27), and (28). As a matter of fact, any localized solutions (or their derivatives) with completely elastic (or not completely elastic or completely inelastic) interaction behaviors of any known (1+1)-dimensional integrable models can be utilized to construct (2+1)-dimensional localized solitonic solutions with completely elastic ($P_i^{+}=P_i^{-}$, $Q_j^{+}=Q_j^{-}$ for all i,j) (or not completely elastic or completely inelastic ($P_i^{+}\neq P_i^{-}$, $Q_j^{+}\neq Q_j^{-}$ at least for one of i,j)) interaction properties. However, to the best of our knowledge, the interactions among semifoldons, peakon, dromions, and compactons were not reported in the literature. In order to see the interaction behaviors among them more direct and visually, we investigate some special examples by fixing the arbitrary functions p and q in equation (19). For convenience, we set $\lambda = a_1 = a_2 = 1$, $a_3 = 0.2$ in equation (19) in the following discussion.

3.2 Completely elastic interactions

Now we discuss some new coherent structures for the physical quantity Φ, and focus our attention on some (2+1)-dimensional semifolded localized structures, which may exist in certain situations, when the function q is t-independent and p is selected via the relations,

$$p_x=\sum_{i=1}^{M} U_i(\xi+w_i t),$$
$$x=\xi+\sum_{i=1}^{M} X_i(\xi+w_i t),$$
$$p=\int^{\xi} p_x x_\xi d\xi, \quad (31)$$

where U_i and X_i are localized excitations with the properties $U_i(\pm\infty)=0, X_i(\pm\infty)=\text{const}$. From equation (31), one can knows that ξ may be a multi-valued function in some suitable regions of x by selecting the functions X_i appropriately. Therefore, the function p_x, which is obviously an interaction solution of M localized excitations because of the property $\xi\left|_{x\to\infty}\right. \to \infty$, may be a multi-valued function of x in these areas, though it is a single-valued functions of ξ. Actually, most of the known multi-loop solutions are a special situation of equation (31). In general terms, if the functions p or q are taken as multiple localized excitations that possess the phase shifts of (1+1)-dimensional models then the (2+1)-dimensional localized excitations involving representation (19) inherit phase shifts structures. As simple choices for the functions p and q one can take,

$$p_x=\sum_{i=1}^{M} U_i(\xi+w_i t),$$
$$x=\xi+\sum_{i=1}^{M} X_i(\xi+w_i t), \quad (32)$$
$$q=1+\sum_{j=1}^{N}\exp\Big[k_j\left(y+\beta_j t\right)+y_{0j}\Big], \quad (33)$$

where k_j, β_j, w_i, and y_{0j} are arbitrary constants and M, N are positive integers. If taking the concrete forms of p and

$$q=\sum_{i=1}^{N}\begin{cases}0, & y+\beta_i t\le y_{0i}-\frac{\pi}{2k_i}\\ b_i\cos^{\alpha_i+1}\Big(k_i(y+\beta_i t-y_{0i})\Big), & y_{0i}-\frac{\pi}{2k_i}<y+\beta_i t\le y_{0i}+\frac{\pi}{2k_i}\ ,\\ 0, & y+\beta_i t>y_{0i}+\frac{\pi}{2k_i}\end{cases} \tag{37}$$

q as follows

$$p_x=\frac{4}{5}\text{sech}^2(\xi)+\frac{1}{2}\text{sech}^2(\xi-0.3t),\ x=\xi-1.5\tanh(\xi)$$
$$-1.5\tanh(\xi-0.3t), \tag{34}$$
$$q=1+\exp(y), \tag{35}$$

then we successfully construct semifolded localized excitations that possess phase shifts for the physical quantity Φ depicted in Figure 1. From Figure 1, we can see that the two semifolded localized excitations possess novel properties, which fold in the y direction, and localize in a usual single valued way in the x direction. Moreover, one can find the interaction between the two semifolded localized excitations (semifoldons) is completely elastic, which is very similar to the completely elastic collisions between two classical particles, since the velocity of one of the localized structures has set to be zero and there are still phase shifts for the two semifolded localized excitations. To see more carefully, one can easily find that the position located by the large static localized structure is altered from about $x=-1.5$ to $x=1.5$ and its shape is completely preserved after interaction.

Along the same line of argument and performing a similar analysis, when p and q are taken as the following forms,

$$p_x=\sum_{j=1}^{M}U_j(\xi+w_jt),$$
$$x=\xi+\sum_{j=1}^{M}X_j(\xi+w_jt), \tag{36}$$

See equation (37) above

where M and N are positive integers, then we may construct another type semifolded localized structures for the physical quantity Φ. For simplicity, we take

$$p_x=\frac{4}{5}\text{sech}^2(\xi)+\frac{1}{2}\text{sech}^2(\xi-0.3t),$$
$$x=\xi-1.5\tanh(\xi)-1.5\tanh(\xi-0.3t), \tag{38}$$

$$q=\begin{cases}0, & y\le-\frac{\pi}{2}\\ \cos^5(y), & -\frac{\pi}{2}\le y\le\frac{\pi}{2}.\\ 0 & y>\frac{\pi}{2}\end{cases} \tag{39}$$

Then we derive a combined localized coherent structure depicted in Figure 2.

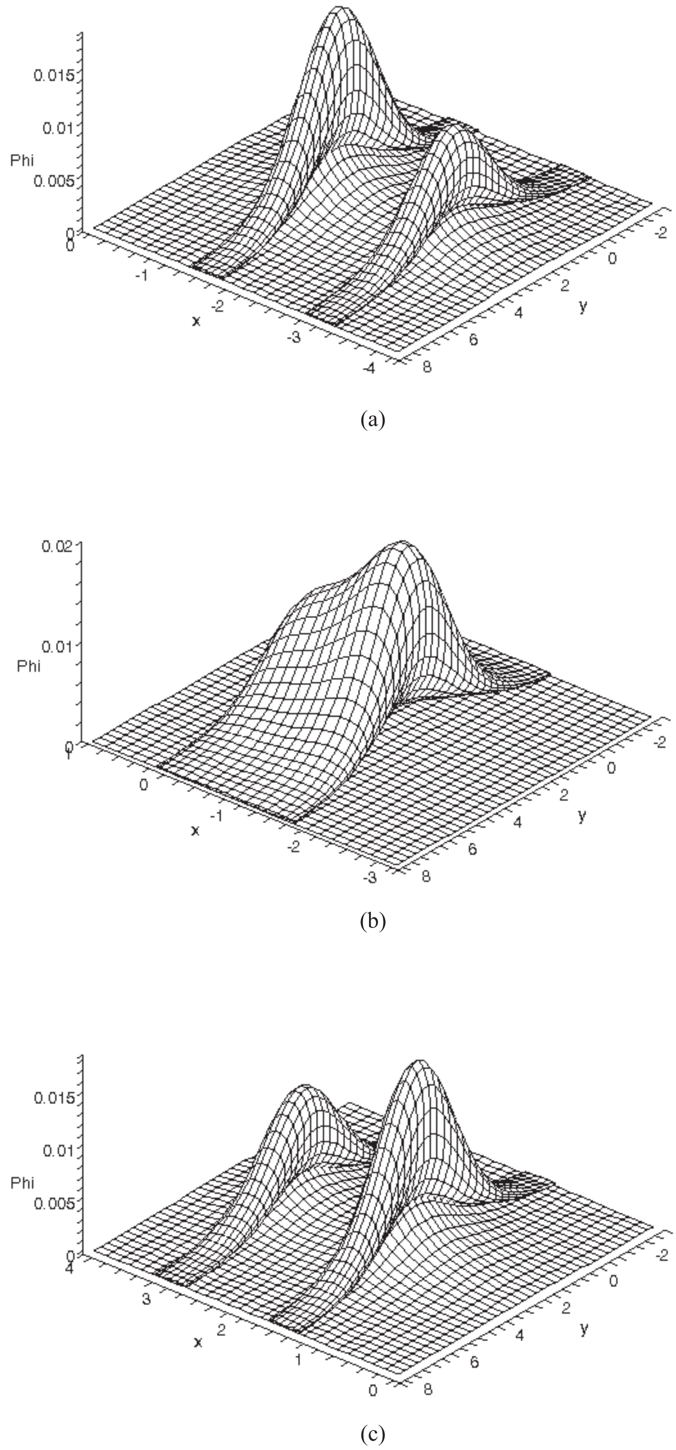
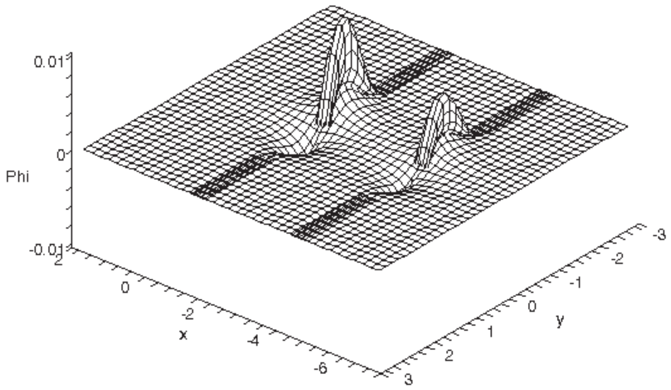
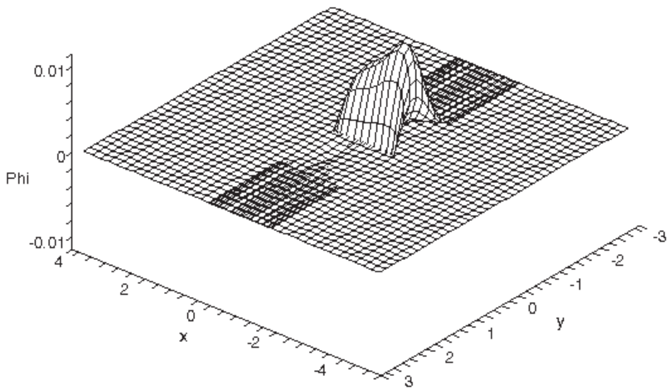


Fig. 1. The evolution of the interactions of two semifolded localized structures for the physical quantity Φ expressed by equation (19) with the conditions (34) and (35) at times (a) $t=-15$, (b) $t=-5$, (c) $t=15$, respectively.

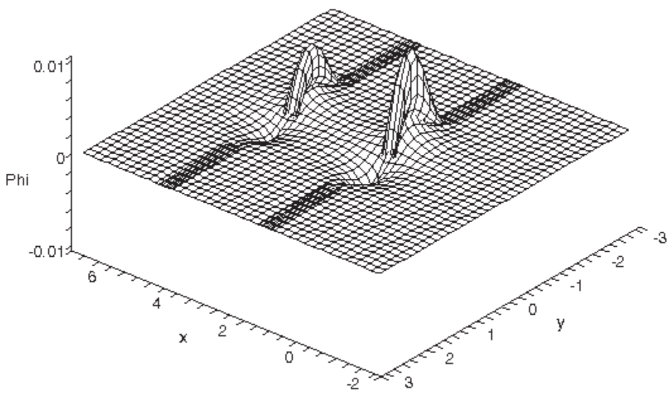
$$q = \begin{cases} \sum_{j=1}^{N} e_j \exp(n_j y + w_j t + y_{0j}), & n_j y + w_j t + y_{0j} \le 0 \\ \sum_{j=1}^{N} \Big(-e_j \exp(-n_j y - w_j t - y_{0j}) + 2e_j\Big), & n_j y + w_j t + y_{0j} > 0 \end{cases}, \tag{41}$$



(a)



(b)



(c)

Fig. 2. The evolution of the interactions of two semifolded localized structures for the physical quantity Φ expressed by equation (19) with the conditions (38) and (39) at times (a) $t = -20$, (b) $t = -5$, (c) $t = 20$, respectively.

According to the above ideas, if we take p and q to have the following forms,

$$p_x = \sum_{i=1}^{M} U_i(\xi + w_i t),$$
$$x = \xi + \sum_{i=1}^{M} X_i(\xi + w_i t), \tag{40}$$

See equation (41) above

where M and N are positive integers, then we may construct third type semifolded localized structures for the physical quantity Φ. For convenience, we select

$$p_x = \frac{4}{5}\mathrm{sech}^2(\xi) + \frac{1}{2}\mathrm{sech}^2(\xi - 0.3t),$$
$$x = \xi - 1.5\tanh(\xi) - 1.5\tanh(\xi - 0.3t), \tag{42}$$

$$q = \begin{cases} \exp(y) & y \le 0 \\ -\exp(-y) & y > 0 \end{cases}, \tag{43}$$

and find that their interaction is also completely elastic. The corresponding plot is depicted in Figure 3.

3.3 Non-completely elastic interactions

It is interesting to mention that though the above choices lead to completely elastic interaction behaviors for the (2+1)-dimensional solutions, one can also derive some combined localized coherent structures with non-completely elastic interaction behaviors by selecting p and q appropriately. One of simple choices of the combined localized coherent structures with non-completely elastic interaction behavior is

$$p_x = \sum_{i=1}^{M} U_i(\xi + w_i t),$$
$$x = \xi + \sum_{i=1}^{M} X_i(\xi + w_i t), \tag{44}$$

$$q = a_0 + \sum_{j=1}^{N} B_j \tanh\Big[K_j\,(y + \beta_j t) + y_{0j}\Big], \tag{45}$$

where $a_0, B_j, K_j, \beta_j, w_i$, and y_{0j} are all arbitrary constants, and M, N are positive integers. We can find that the interaction between semifoldons and dromions may exhibit a novel property, which is non-completely elastic since their shapes are not completely preserved after interaction. In order to clarify this phenomenon more clearly

$$q = \begin{cases} a_0, & y + \beta_i t \leq y_{0i} - \frac{\pi}{2k_i} \\ a_0 + \sum\limits_{i=1}^{N} \Big(b_i \sin\left(k_i(y + \beta_i t - y_{0i})\right) + b_i \Big), & y_{0i} - \frac{\pi}{2k_i} < y + \beta_i t \leq y_{0i} + \frac{\pi}{2k_i} \\ a_0 + \sum\limits_{i=1}^{M} 2b_i, & y + \beta_i t > y_{0i} + \frac{\pi}{2k_i} \end{cases}, \tag{49}$$

$$q = \begin{cases} \sum\limits_{j=1}^{N} e_j \exp(n_j y + w_j t + y_{0j}), & n_j y + w_j t + y_{0j} \leq 0 \\ \sum\limits_{j=1}^{N} \Big(-e_j \exp(-n_j y - w_j t - y_{0j}) + 2e_j \Big), & n_j y + w_j t + y_{0j} > 0 \end{cases}, \tag{53}$$

and visually, an example is depicted in Figure 4 when the related functions are fixed as follows

$$\begin{aligned} p_x &= \frac{4}{5}\mathrm{sech}^2(\xi) + \frac{1}{2}\mathrm{sech}^2(\xi - 0.3t), \\ x &= \xi - 1.5\tanh(\xi - 0.3t), \end{aligned} \tag{46}$$

$$q = \tanh(y). \tag{47}$$

Another example is provided by a combined semifoldon and compacton soliton solutions in the (2+1)-dimensional system. The corresponding ansatz is

$$p_x = \sum_{i=1}^{M} U_i(\xi + w_i t), \quad x = \xi + \sum_{i=1}^{M} X_i(\xi + w_i t), \tag{48}$$

See equation (49) above

where M and N are positive integers, then we may construct another non-completely elastic interaction example for the physical quantity Φ. For simplicity, we choose

$$\begin{aligned} p_x &= \frac{4}{5}\mathrm{sech}^2(\xi) + \frac{1}{2}\mathrm{sech}^2(\xi - 0.3t), \\ x &= \xi - 1.5\tanh(\xi - 0.3t), \end{aligned} \tag{50}$$

$$q = \begin{cases} 0, & y \leq -\frac{\pi}{2} \\ \sin(y) + 1, & -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}, \\ 2 & y > \frac{\pi}{2} \end{cases} \tag{51}$$

and can derive a combined semifoldon-compacton localized coherent structure with non-completely elastic behavior depicted in Figure 5.

In fact, we can also construct combined semifoldon-peakon localized coherent structures with non-completely elastic interaction behaviors by selecting p and q as

$$\begin{aligned} p_x &= \sum_{i=1}^{M} U_i(\xi + w_i t), \\ x &= \xi + \sum_{i=1}^{M} X_i(\xi + w_i t), \end{aligned} \tag{52}$$

See equation (53) above

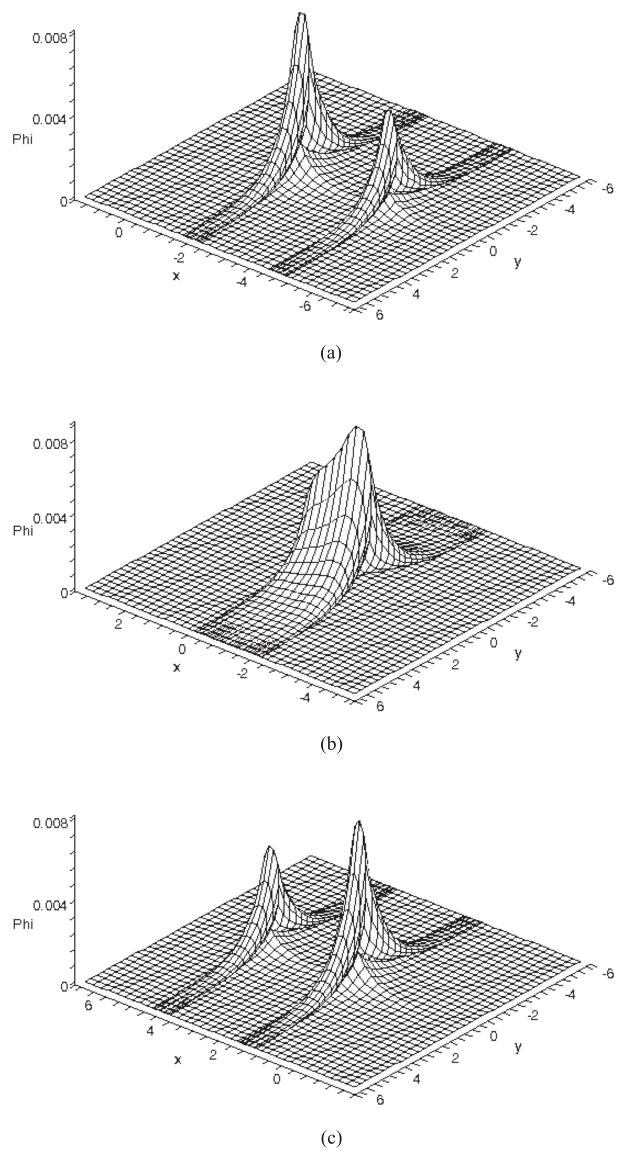


Fig. 3. The evolution of the interactions of two semifolded localized structures for the physical quantity Φ expressed by equation (19) with the conditions (42) and (43) at times (a) $t = -20$, (b) $t = -5$, (c) $t = 20$, respectively.

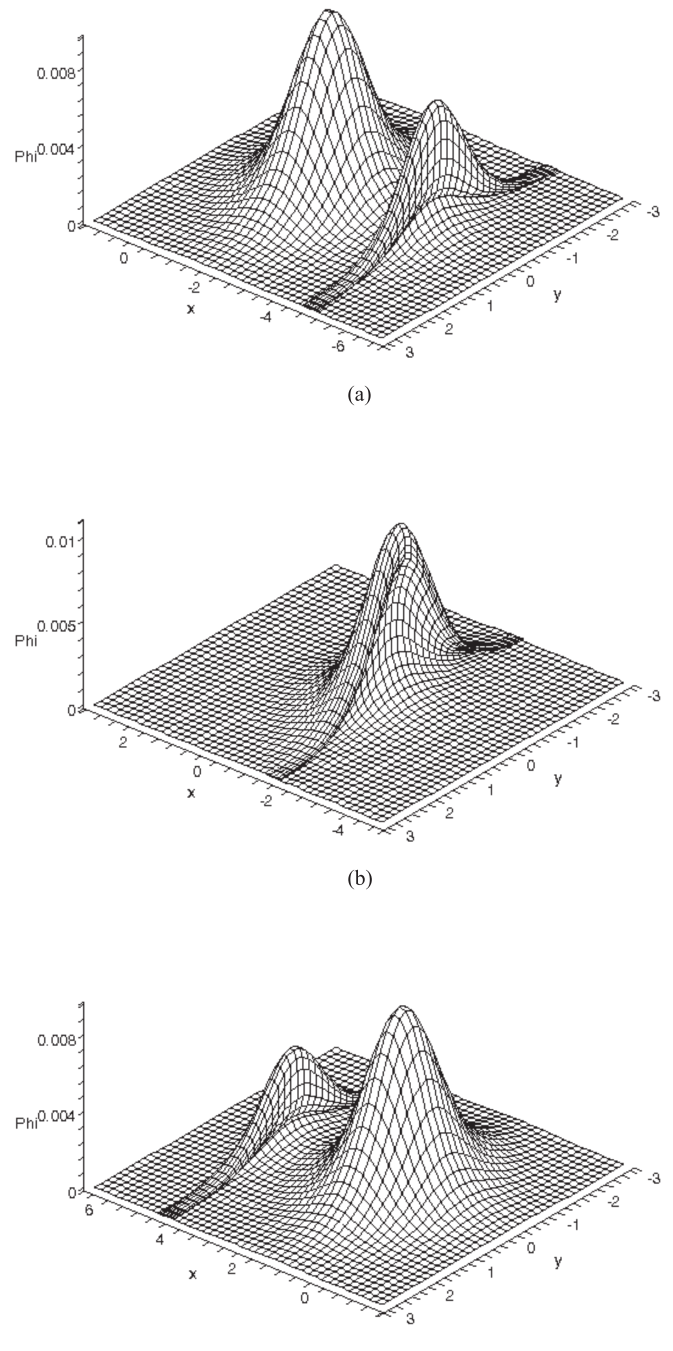


Fig. 4. The evolution of the interactions between semifoldon and dromion for the physical quantity Φ expressed by equation (19) with the conditions (46) and (47) at times (a) $t = -15$, (b) $t = -5$, (c) $t = 15$, respectively.

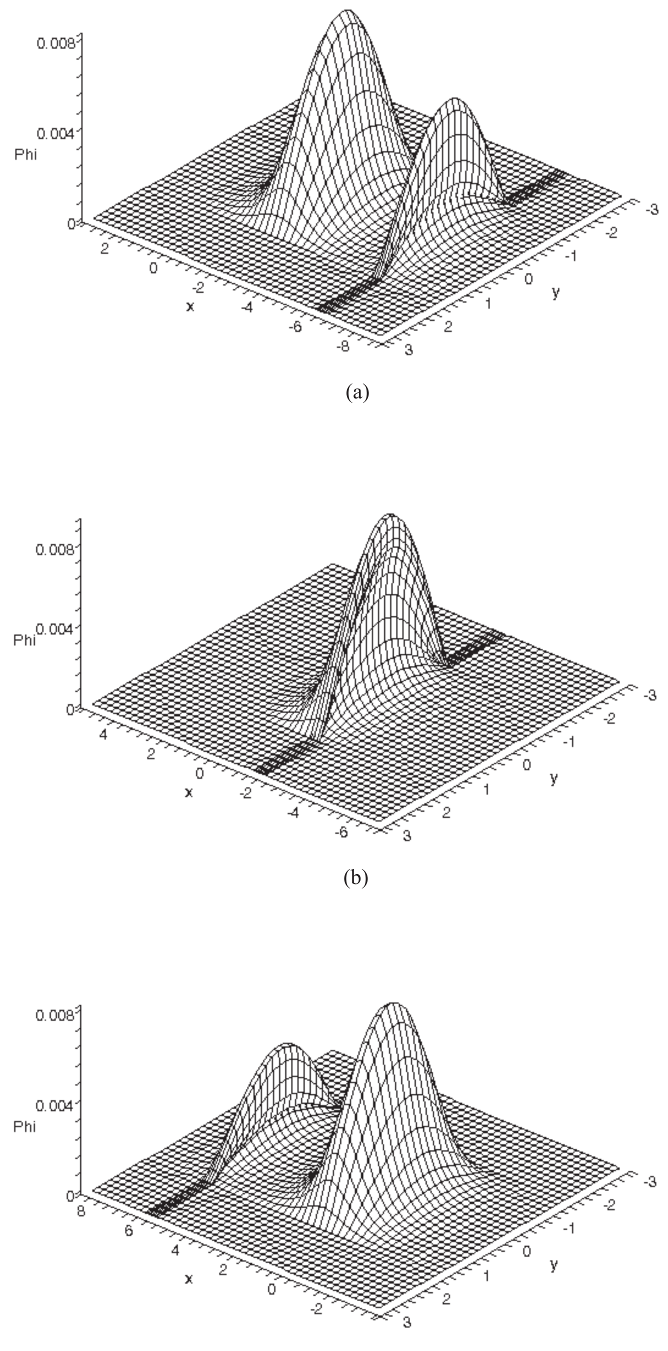


Fig. 5. The evolution of the interactions between semifoldon and compacton for the physical quantity Φ expressed by equation (19) with the conditions (50) and (51) at times (a) $t = -20$, (b) $t = -5$, (c) $t = 20$, respectively.

where M and N are positive integers. Because of the complexity, here we just write down the simplest case

$$p_x = \frac{4}{5}\text{sech}^2(\xi) + \frac{1}{2}\text{sech}^2(\xi - 0.3t),$$

$$x = \xi - 1.5\tanh(\xi - 0.3t), \quad (54)$$

$$q = \begin{cases} \exp(y) & y \leq 0 \\ -\exp(-y) & y > 0 \end{cases}, \quad (55)$$

The corresponding evolution plot is displayed in Figure 6.

4 Summary

Starting from the obtained variable separated excitations, which describe some quite universal (2+1)-dimensional

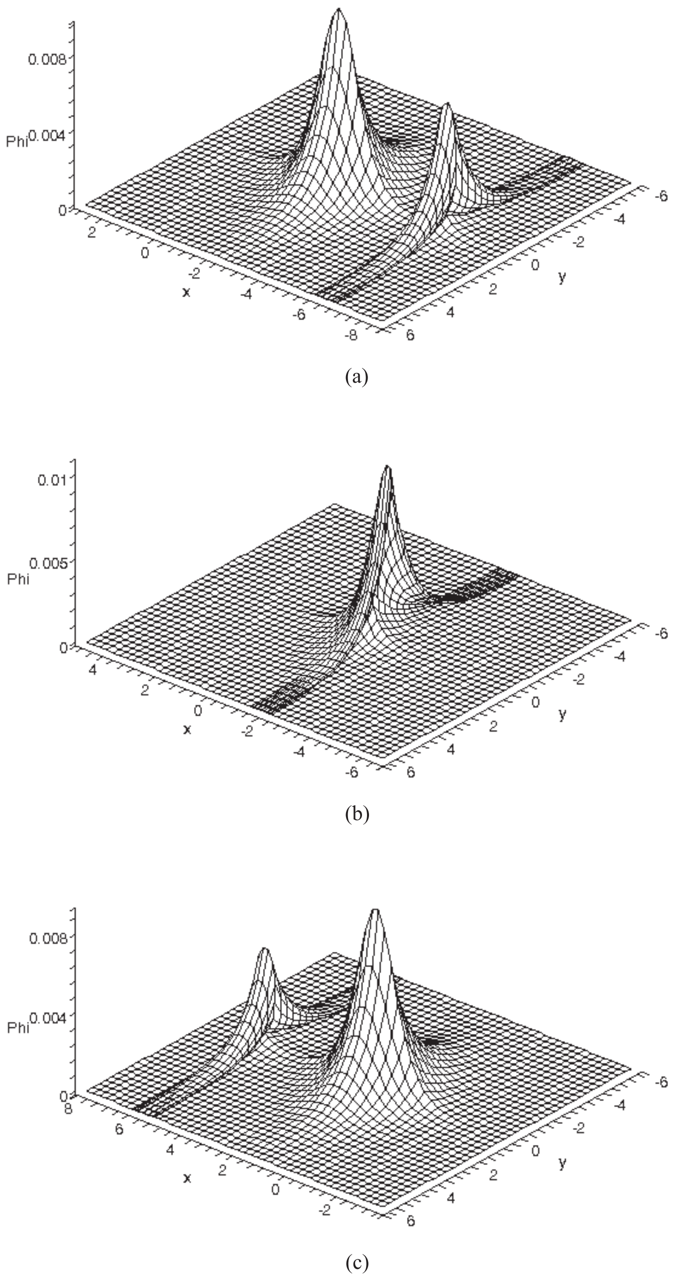


Fig. 6. The evolution of the interactions between semifoldon and peakon for the physical quantity Φ expressed by equation (19) with the conditions (54) and (55) at times (a) $t = -20$, (b) $t = -5$, (c) $t = 20$, respectively.

physical model, of a (2+1)-dimensional system, we discuss the interactions among semifoldons, peakons, dromions, and compactons both analytically and graphically, and reveal some novel properties and interesting behaviors: the interactions among semifoldons are completely elastic that possess phase shifts and the interactions of semifoldon-dromion, semifoldon-compacton, and semifoldon-peakon are non-completely elastic depending on the specific details of the solutions. Because of the complexity of folded phenomena and the wide applications of the soliton theory, to learn more about the new localized structures and interactions between different types of solitary waves and their applications in reality is worth further study.

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